comparative advantage acts as an agglomeration force given that it has an intersectoral nature and favours a sustainable core-periphery outcome. We note what follows

Proposition 8 *When the manufacturing sector is agglomerated in the region with a technological disadvantage, an increase of trade costs enhances agglomeration if trade costs are small (ϕ is low) and dispersion if trade costs are intermediate (ϕ is intermediate). However, when the manufacturing sector is agglomerated in the region with a technological advantage, an increase of trade costs may only reduce agglomeration.*

Previous proposition recalls the results by Ricci ([13], p. 367), who, in a different framework, obtains that "if the large country has a comparative disadvantage, a rise in trade costs may enhance agglomeration".

Finally, we have not so far considered how the productivity differential gap is determined. One determinant could be the existence of geographically localized spillovers which may produce higher productivity levels in the region in which all skilled workers are concentrated. However, if a too high concentration of workers creates some problems of coordination in the organization of the production process, then this kind of congestion force at work would reverse the technological gap in favour of the region with the lowest concentration of workers.

4 Symmetric equilibrium stability

In this section we reclassify centripetal and centrifugal forces with respect to the symmetric equilibrium in order to take into account the fact that technological differences may exist. To evaluate the intensity of centripetal and centrifugal forces in the symmetric equilibrium we rewrite expression (15) in the following way:

$$R_r = w_{hr} = a_r^{\frac{\rho}{1-\rho}} \mu \left[\left(\frac{w_{hr}}{p_{mr}} \right)^{-\frac{\rho}{1-\rho}} (w_{hr} H_r + L) + \left(\frac{w_{hr}}{p_{mv}} \right)^{-\frac{\rho}{1-\rho}} \phi \left(w_{hv} H_v + L \right) \right] \tag{33}$$

R_r are equilibrium sales of a firm in region r, and an analogous expression, R_v, can be obtained for region v. We evaluate R_r in order to define different centripetal and centrifugal forces that are in action when the two regional economies are in the neighborhood of the symmetric equilibrium. Particularly, as in the previous section, we distinguish between fixed-technology (or traditional forces) and variable-technology (or non-traditional forces).[12] Starting from a symmetric equilibrium, if a technological gap arises, it needs to be closed in order to allow a return to the same. An initial departure from the perfectly symmetric situation described in the symmetric equilibrium, gives rise to traditional and less traditional agglomeration and dispersion forces. *Traditional* forces are identified by Baldwin et al. [1] as follows: two agglomeration forces, respectively called *market-access* and *cost of living effects*, and one dispersion force, the so-called *market-crowding effect*. These forces continue to act in our version of the model, and they may be commented following Baldwin et al. [1]. However, in our version, technological differences may add a further agglomeration force because they drive away the two economies from the symmetric equilibrium. It is, in fact, particularly easy to verify that when, for instance, a_r increases with respect to a_v, for given values of other factors in R_r and R_v, R_r increases with respect to R_v giving firms the incentive to move from region v to region r, increasing labor demand in r and, in so doing, encouraging a more intensive migration toward this region. We notice that the intensity of these forces increases for higher values of μ.

In order to study symmetric equilibrium stability in greater detail, we must remember that it requires all variables, endogenous and exogenous, and all parameters to be equal in the two regions. Specifically, from a technological point of view, this requires that $a_r = a_v = a$. Moreover, following Fujita et al. [4], we recall that in the neighborhood of the symmetric equilibrium, changes in the value of a regional variable are associated with changes of the same amount, but of the inverse sign, in the correspondent variable in the other region. For instance, a change in the number of skilled workers in a region, dH_r, is associated with the change $dH_v = -dH_r$ in the other region. This is still valid in our simple extension of the standard model, where we also need to consider that regional productivity level changes, described by (19), depend on the interregional distribution of skilled workers. It is easily verified that in the neighborhood of the symmetric equilibrium $da_r = -da_v$.

[12] Expression (33) is useful for comparing our results with the ones presented in Baldwin et al. [1].

From the choice of the numèraire and from the assumption on the traditional good, we know that

$$p_{ar} = p_{av} = w_{lr} = w_{lv} = 1 \tag{34}$$

After substituting prices from (11), we derive the first order Taylor expansion in the neighborhood of the symmetric equilibrium for: each manufacturing variety produced in both regions (15), the manufacturing price indexes (4), skilled workers' real wages (17), and total regional incomes (18). The resulting expressions are used to derive, after a number of appropriate substitutions, expression (35):

$$\begin{aligned} d\varpi_h &= \frac{2Zp_m^{-\mu}}{(1-\sigma)^2\Delta'}\left\{\mu(1-2\sigma) - Z\left[1-\sigma\left(1+\mu^2\right)\right]\right\}dH + \\ &\quad + \frac{p_m^{-\mu}}{(1-\sigma)\Delta'}\left\{Z^2\left(1-\sigma-\mu^2\right) + Z\mu + (\sigma-1)\right\}\frac{da}{a} \end{aligned} \tag{35}$$

where $Z = \frac{(1-\phi)}{(1+\phi)}$ and $\Delta' = \frac{(1-\sigma)Z^2 - Z\mu + \sigma}{(1-\sigma)}$. Z is an index of the "closedness" of trade with its value that range from 0, with free trade to 1, with autarchy. Expression (35) shows how regional real wages changes in the neighborhood of the symmetric equilibrium, $d\varpi_h$, depend on changes in the regional number of skilled workers dH, and on changes in technological development levels, da.

We have already noticed that in correspondence to the symmetric equilibrium we have: $a_r = a_v = a$ and we normalize $a = 1$. Therefore (35) can be rewritten as follows

$$\begin{aligned} d\varpi_h &= \frac{p_m^{-\mu}}{(1-\sigma)^2\Delta'}\{\left[2Z\mu(1-2\sigma) - 2Z^2\left(1-\sigma\left(1+\mu^2\right)\right)\right]dH + \\ &\quad +(1-\sigma)[Z^2\left(1-\sigma-\mu^2\right) + Z\mu + (\sigma-1)]da\} \end{aligned} \tag{36}$$

Finally, from the first order Taylor expansion in the neighborhood of the symmetric equilibrium for equation (20), we get

$$da = \kappa dH \tag{37}$$

Substituting (37) in (36) gives

$$\begin{aligned} d\varpi_h &= \frac{p_m^{-\mu}}{(1-\sigma)^2\Delta'}\{\left[2Z\mu(1-2\sigma) - 2Z^2\left(1-\sigma\left(1+\mu^2\right)\right)\right] + \\ &\quad +(1-\sigma)[Z^2\left(1-\sigma-\mu^2\right) + Z\mu + (\sigma-1)]\kappa\}dH \end{aligned} \tag{38}$$

Finally, we recall that $\rho = (\sigma-1)/\sigma$, and we rewrite (38) as follows

$$\begin{aligned} \frac{d\varpi_h}{dH} &= \frac{p_m^{-\mu}(1-\rho)}{\rho\left[1-(1-\rho)Z\mu-\rho Z^2\right]}\{2Z\left[\mu(1+\rho) - Z\left(\rho+\mu^2\right)\right] + \\ &\quad -\frac{\rho}{1-\rho}[Z^2\left(\rho+\mu^2(1-\rho)\right) - (1-\rho)Z\mu - \rho]\kappa\} \end{aligned} \tag{39}$$

The symmetric equilibrium is stable if $d\varpi_h/dH$ is less than 0, and on the contrary it is unstable if it is greater than (or equal to) 0. We observe that the sign of $p_m^{-\mu}(1-\rho)/\rho$ in (39) is always positive. Hence, the sign of $d\varpi_h/dH$ depends on the sign of two terms: the denominator, which we call d, and the term in curly brackets, which we call g. These terms are respectively

$$d = 1-(1-\rho)Z\mu - \rho Z^2 \tag{40}$$

and

$$g = g_0 - g_1 \tag{41}$$

with

$$g_0 = 2Z\left[\mu(1+\rho) - Z\left(\rho+\mu^2\right)\right] \quad \text{and} \quad g_1 = \frac{\rho}{1-\rho}[\left(\rho+\mu^2(1-\rho)\right)Z^2 - (1-\rho)\mu Z - \rho]\kappa \tag{42}$$

We can recall that parameters μ, ρ and Z assume values contained in the range $[0,1]$. We consider how different values of Z determine the sign of d and g, which jointly affect the sign of $d\varpi_h/dH$, and, therefore, the stability properties of the symmetric equilibrium.

We note that for given values of μ and ρ, d is a parabola that opens downward, whose graphic is given in figure 3, where only the relevant range of Z values, that is $0 \le Z \le 1$, must be considered.

Insert figure 3 about here

If $Z = 0$, then $d = 1$. d has its maximum value for a negative value of Z. Hence, for $Z \in [0, 1]$, d is decreasing and its intercept with the axis of abscissas is $Z_d = \frac{-\mu(1-\rho)+\sqrt{\mu^2(1-\rho)^2+4\rho}}{2\rho}$. It is easy to show that $Z_d > 1$, given that for $Z = 1$, $d = (1-\mu)(1-\rho) > 0$.

Finally, to complete the study of the sign of $d\varpi_h/dH$, we need to discern two cases: case I and case II. In case I, a higher share of skilled workers in a region does not imply a higher (or lower) productivity level. In other words, there are no geographically localized technological externalities, and, thus, $\kappa = 0$. Instead, in case II, κ can be positive due to the existence of a technological positive (negative) externality that implies a higher (lower) productivity level of the region in which the skilled worker' share is higher. In particular, as it was stressed in the introduction, we will concentrate on the study in which, if there exists any technological externality, this is of the positive type, with $\kappa > 0$.

4.1 Case I: $\kappa = 0$.

When geographically localized technological externalities do not exist, that is when $\kappa = 0$, g coincides with g_0 given in expression (42). In the plane (g_0, Z), g_0 is a parabola that opens downward, and its graphic is represented in figure 4a.

Insert figure 4a about here

The intercepts of g_0 with the Z-axis are $Z = 0$ and $Z_0 = \frac{\mu(1+\rho)}{(\rho+\mu^2)} > 0$, while its maximum occurs at $Z = \frac{\mu(1+\rho)}{2(\rho+\mu^2)}$.

It can be proved that if $\rho > \mu$, that is if the "no black hole condition" identified by Fujita et al. [4] holds, then $Z_0 < 1$. Thus, we may enunciate the following proposition.

Proposition 9 *If $\kappa = 0$, the symmetric equilibrium is stable for $Z \in (Z_0, 1]$; it is unstable for $Z \in [0, Z_0]$, with $Z_0 = \frac{\mu(1+\rho)}{(\rho+\mu^2)}$.*

This means that the symmetric equilibrium is stable only if trade costs are high and the degree of integration is small, while it is unstable for low trade costs. These results are in line with those by Fujita et al. [4]. Moreover, we may remark that if $\kappa = 0$ and $a = 1$, expression (39) coincides with the expression by Fujita et al. [4] at page 73

$$\frac{d\varpi_h}{dH} = 2Zp_m^{-\mu}\left(\frac{1-\rho}{\rho}\right)\left[\frac{\mu(1+\rho) - Z\left(\rho + \mu^2\right)}{1 - Z\mu(1-\rho) - \rho Z^2}\right] \tag{43}$$

and, in this case, the symmetric equilibrium unstable for $Z \in [0, Z_0]$ and stable for $Z \in (Z_0, 1]$, with the break point level of trade costs $\tau^{\rho/(1-\rho)} = \frac{(\rho+\mu)(1+\mu)}{(\rho-\mu)(1-\mu)}$. In this work we show that these ranges change when $\kappa > 0$.

4.2 Case II: $\kappa > 0$

When positive geographically localized technological externalities exist, that is when $\kappa > 0$, in expression (39) g is given by g_0 minus g_1 in expression (42). We have already discussed the properties of g_0. g_1 is a parabola that opens upward. The intercepts of g_1 with the Z-axis are $Z = Z_1$ and $Z = Z_2$. We notice that $Z_1 > 1$ and $Z_2 < 0$, given that the following results hold: $g_1 = (\mu - 1)\,\mu\rho\kappa < 0$ when $Z = 1$; $g_1 = -\frac{\rho^2}{1-\rho}\kappa < 0$ when $Z = 0$, and that the minimum of g_1 is for $Z \in [0, 1]$. In fact, the slope of the parabola is negative (that is $-(1-\rho)\mu < 0$) when $Z = 0$, and it is positive (and equal to $\rho\,(1+\mu) + \rho - \mu + 2\mu^2(1-\rho) > 0$) when $Z = 1$.[13]

The graphic of g_1 is represented in figure 4b, where only $Z \in [0, 1]$ are the relevant values of the closedness of trade.

Insert figure 4b about here

We must remember that in this section we consider only parameter values for which the “no black hole condition” holds, with $\rho > \mu$.

Comparing g_1 with g_0 it is possible to define the sign of expression g in (39). g is always positive (negative) when Z is such that $g_0 > g_1$ ($g_0 < g_1$). We know that g_0 and g_1 cross only once when Z is positive, when $Z = Z^*$. Therefore, we may state that g is positive (negative) when $0 \leq Z < Z^*$ ($Z > Z^*$). However, we notice that Z^* can be higher or lower than 1. Clearly,

[13] Note that the “no black hole condition” identified by Fujita et al. [4] holds with $\rho > \mu$.

we are interested in defining the sign of g only for $Z \in [0,1]$, that is for the relevant values of the closedness of trade.

Proposition 10 *When $Z^* < 1$, the symmetric equilibrium is stable for $Z \in (Z^*, 1]$ and unstable for $Z \in [0, Z^*]$. When $Z^* \geq 1$, the symmetric equilibrium is always unstable.*

Consequently, to avoid the case in which the symmetric equilibrium is unstable for every value of Z, a new condition must be stated that we call "pro dispersion condition" and that will be explicitly defined in the following pages.

First, we want to underline that given the shape of the two parabolas g_0 and g_1, their intersection in Z^* may identify two subcases, respectively denoted A and B, according to the values of parameters in the model.[14]

Case 11 (A) *If parameter values are such that $g_0 < g_1$ when $Z = 1$, then $Z^* < 1$.*

Case 12 (B) *If parameter values are such that $g_0 > g_1$ when $Z = 1$, then $Z^* > 1$.*

Specifically, we must establish when $Z^* \lesseqgtr 1$. The sign of the inequality depends on the value of κ, the index of the geographically localized technological externalities. A necessary and sufficient condition that must be satisfied in order to have a range of Z values for which the symmetric equilibrium is stable is that $g_1 > g_0$ when $Z = 1$. It can be readily verified that $g_1(Z = 1) > g_0(Z = 1)$ if

$$\kappa < \kappa^* = \frac{2(\rho - \mu)}{\mu \rho} \tag{44}$$

Therefore a range of Z values for which the symmetric equilibrium is stable does exist, only if

$$\kappa < \kappa^* \text{ with } \kappa^* > 0 \tag{45}$$

Therefore, the above mentioned "pro dispersion condition" must hold with $\kappa < \kappa^*$ in order to have at least some value of trade costs for which the symmetric equilibrium is stable. Figure 5 shows the case in which the symmetric equilibrium may be stable (because $\kappa < \kappa^*$), while figure

[14] To compare our results with those by Krugman [6] and [7], we remind that we consider this traditional no black hole condition because we start from the point in which $a_r = a_v = 1$.

6 shows the case in which the symmetric equilibrium is always unstable (because $\kappa > \kappa^*$).

Insert figure 5 about here

Insert figure 6 about here

Comparing the sign of expression g with that of d, we may write what follows.

Proposition 13 *When $0 < \kappa < \kappa^*$, $Z^* < 1$ exists and the symmetric equilibrium is stable when $Z \in (Z^*, 1]$ (because $\frac{d\varpi_h}{dH} < 0$), and unstable when $Z \in [0, Z^*]$ (because $\frac{d\varpi_h}{dH} \geq 0$).*

Proposition 14 *When $\kappa > \kappa^*$, the symmetric equilibrium is unstable $\forall Z \in [0, 1]$ given that $Z^* > 1$ (because $\frac{d\varpi_h}{dH} > 0$).*

The range of Z for which the symmetric equilibrium is stable when $\kappa > 0$ is smaller than that for $\kappa = 0$. We may compare the ranges that we obtain for $\kappa = 0$ with those that correspond to $\kappa > 0$ and write the following proposition.

Proposition 15 *In general, when we consider the symmetric equilibrium and the productivity level is positively related to skilled workers density in the neighborhood of the symmetric equilibrium ($\kappa > 0$), the range of Z for which the symmetric equilibrium is stable is smaller than in the case in which the positive externality does not exist ($\kappa = 0$).*

We may comment on this result considering expression (33). The migration of a certain number of skilled workers move regional economies from the symmetric equilibrium in its neighborhood. Let us consider, for instance, the case in which a certain number of skilled workers moves from region v to region r. In this case the centrifugal force generated by the market crowding effect in region r is weaker than centripetal forces. Indeed, prices in the larger market r diminish because its productivity increases and skilled workers real wages increase in r, strengthening the intensity of centripetal forces and reducing the range of Z for which the symmetric equilibrium is stable. Thanks to the geographically localized externality generated when $\kappa > 0$, the intensity of centripetal forces increases with respect to the centrifugal one. Hence, the width of Z for which the symmetric equilibrium is stable is reduced with respect to the case in which this externality does not exist ($\kappa = 0$). The technological externality produced by a positive κ value does strengthen the variable-technology centripetal force reducing the width of the range of Z values for which the symmetric equilibrium is stable.

We notice that when κ increases, the width of the range for which the symmetric equilibrium is stable decreases. Moreover, it can be readily verified that

$$\frac{\partial \kappa^*}{\partial \mu} < 0 \qquad \text{and} \qquad \frac{\partial \kappa^*}{\partial \rho} > 0$$

Therefore, the symmetric equilibrium is more likely to be stable, the smaller the share of expenditures in manufacturing and the degree of product differentiation are.

Finally, let us show relative real wages as a function of workers share in region r, to simulate possible outcomes for different levels of trade costs. For this exercise we need to specify how productivity levels depend on workers density, and we use the following equation

$$a_r = 1 + [bH_r(1 - H_r) + c]\,(Hr - \frac{1}{2}) \tag{46}$$

with $b \geq 0$ and c that are shape parameters.[15] Figures 7a and 7b plot relative real wage (premium) in the two regions as a function of the workers' share in region r, when $b > 0$ and $c = 0$. Figure 7a represents the case in which $\tau = 2.11$ and shows that the symmetric equilibrium is stable either when $\kappa = 0$ or $\kappa > 0$. However, if trade costs are smaller with $\tau = 2$, the symmetric equilibrium is still stable in the case of no geographically localized externalities, but it is unstable with positive externalities (Figure 7b).[16]

Insert figures 7a,b about here

[15] Expression (46) is an ad hoc equation that has the following properties: if skilled workers are uniformly distributed between the two regions, regional manufacturing productivity levels are equal, that is $a_r = a_v = 1$. However, if a certain number of skilled workers migrate towards one of the two regions, let say for instance the north, the northern productivity level becomes higher than 1, while that of the south smaller than 1. Specifically, to have these results, $\kappa = b/4 + c$ must be positive. These assumptions reflect the fact that labor productivity becomes higher, the higher the number of skilled workers in that particular region is. This fact reflects positive geographically localized externalities. Increasing the number of skilled workers in a region, increases the density of these workers in the same region and, therefore, may increase knowledge spillovers among the same workers, and, in turn, this increases regional productivity. For this particular goal, c would be sufficient, and b could vanish. Moreover, if c were negative, the externality would be negative describing a congestion effect. However, this could not always be the case, given that productivity levels may decrease also when $\kappa > 0$, only if the number of workers becomes too high (at a level of $H_r > 1/2$). This phenomenon may take place because when the number of skilled workers in a regions becomes too high, it becomes more difficult to coordinate their production activity, or because congestion processes would reduce productivity levels. This is captured by coefficient $b > 0$. The range of admissible values for c is $(-2, 2)$ to avoid negative values of a_r.

Finally, when c is negative, and b is such that $\kappa = b/4 + c > 0$, congestions effects (described by c) may become so high that they involve a productivity level smaller than 1 when mobile workers are completely concentrated in a region. (See, for instance, figure 8)

[16] Figures 7a and 7b are drawn for: $b = 0.2$; $\sigma = 3.33$; $\mu = 0.3$.

Finally, if c is negative, and b is such that the geographically localized externality is positive in the neighborhood of the symmetric equilibrium, that is $\kappa = b/4 + c > 0$, congestions effects may become so strong to imply a productivity level smaller than 1 when mobile workers are completely concentrated in one region. In this case, figure 8 shows that while the symmetric equilibrium is stable for high trade costs ($\tau = 6$), it is unstable for lower trade costs ($\tau = 3$ and $\tau = 2$).[17] Moreover, due to the existence of strong congestion effects, full agglomeration is never stable, while two asymmetric equilibria may be stable.

Insert figure 8 about here

5 Conclusion

This work re-examines Krugman model properties when interregional productivity differences may arise in the modern sector. This reassessment is achieved by means of the description of the intensities of centripetal and centrifugal forces which determine the sustainability of the full agglomeration equilibria of the modern sector.[18] We show how different parameters of the model concur to determine centripetal and centrifugal forces intensities, either in the case of "fixed-technology" or traditional forces, or in the case of "variable-technology" forces.

Moreover, our modified version of the standard economic geography model confirms the finding by Venables [15] that is with Ricardian differences there could exist equilibria characterized by the localization of sectors in the region in which they have a comparative disadvantage, even tough this could happen only for intermediate trade costs. However, we find that when the two regions are sufficiently integrated, the comparative advantage dominates and production localization reflects the comparative advantage with manufacturing production agglomerated in the more productive region, while the agricultural good is produced in both regions. A similar result is obtained by Forslid and Wooton ([3], p. *) who find that "when trade barriers are sufficiently low, comparative

[17] Figure 8 is drawn for: $\sigma = 3.33$; $\mu = 0.3$; $b = 9$; $c = -1$.

[18] Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud [1] stress that the evaluation of agglomeration and dispersion forces in fully agglomerated equilibria is rather difficult.

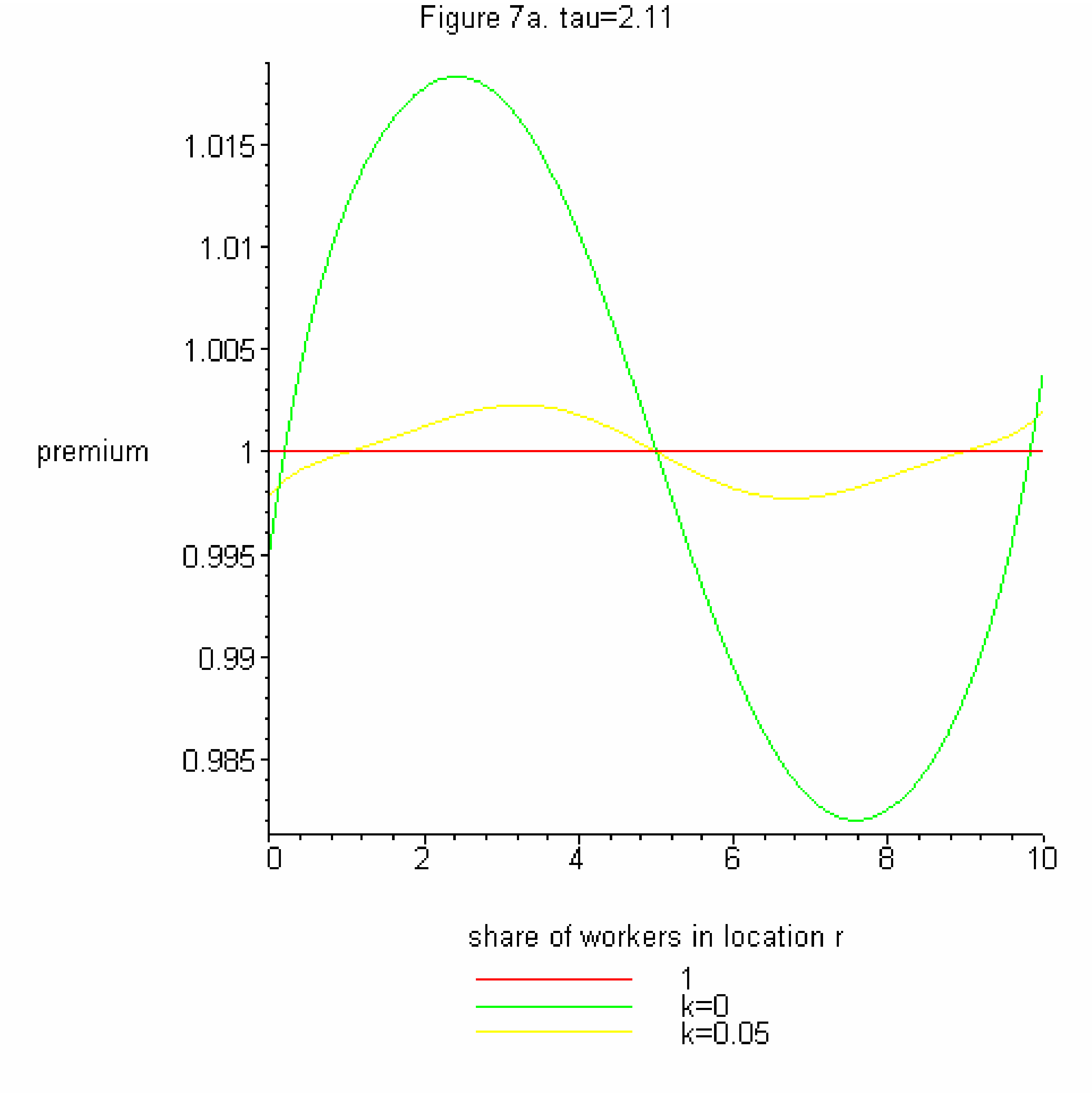
Figure 7a. tau=2.11
premium
1.015
1.01
1.005
1
0.995
0.99
0.985
0
2
4
6
8
10
share of workers in location r
1
k=0
k=0.05

Figure 7b. tau=2

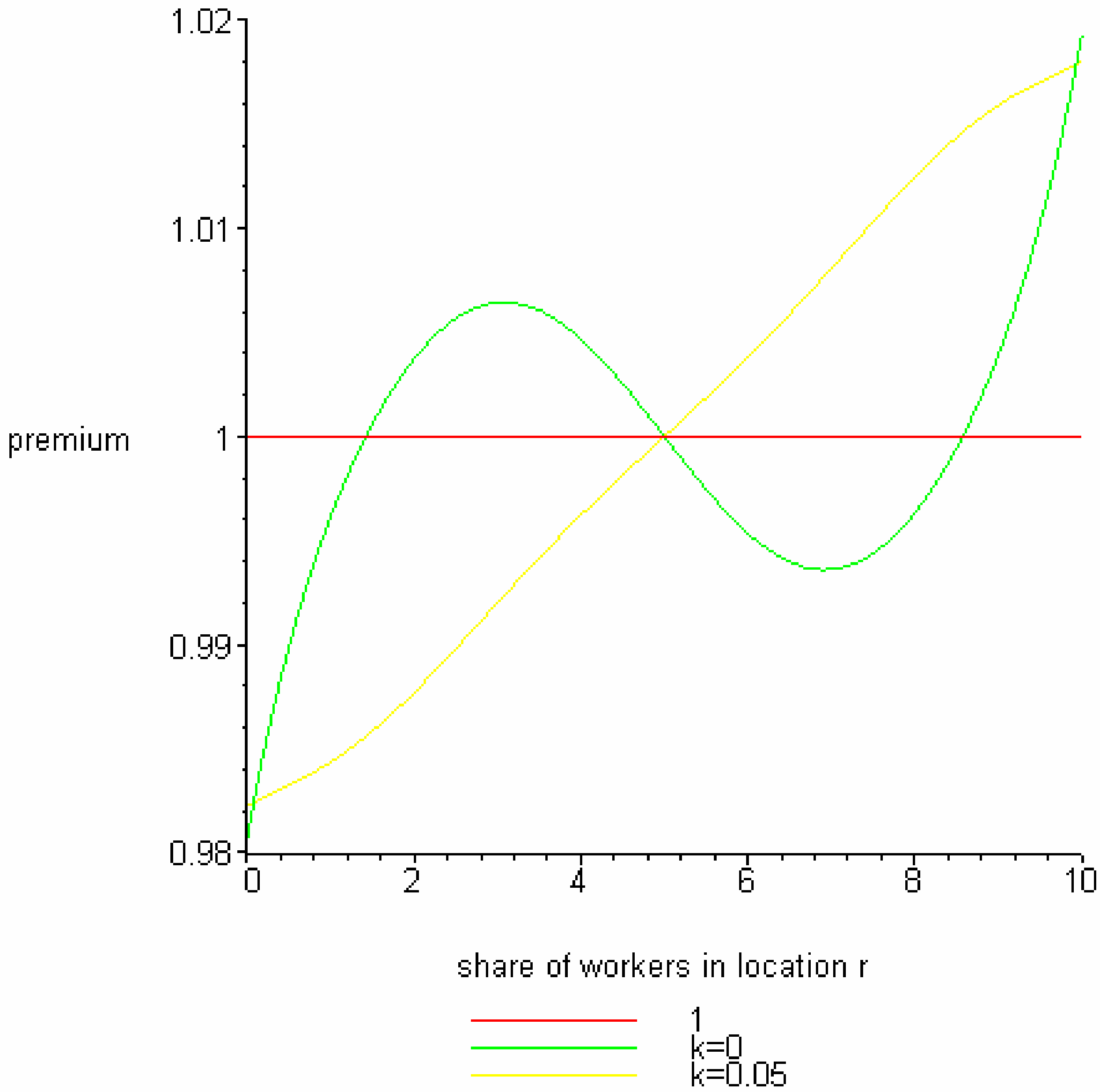

Figure 8, tau=2, tau=4, tau=6